

## ON THE ADVANTAGES OF USING THE SCALED FIELD $f$ AS THE AUXILIARY PARAMETER IN MURPHY-GOOD FIELD EMISSION THEORY

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The Murphy-Good zero-temperature field electron emission (FE) equation for local emission current density  $J$ , as a function of the local work function  $\phi$  and the magnitude  $F$  of the local surface electrostatic (ES) field, can be written in two alternative (but numerically equivalent) mathematical ways. In the "legacy" (or "BKH") approach, the zero-temperature MG FE equation is written

$$J^{\text{MG0}}(\text{BKH}) = \{t_{\text{BKH}}(y_{\text{F}})\}^{-2} \cdot a\phi^{-1}F^2 \exp[-v_{\text{BKH}}(y_{\text{F}}) \cdot b\phi^{3/2}/F], \quad (1)$$

where  $a$  and  $b$  are the Fowler-Nordheim universal constants,  $y_{\text{F}}$  is the Nordheim parameter defined below, and  $v_{\text{BKH}}(y_{\text{F}})$  and  $t_{\text{BKH}}(y_{\text{F}})$  are functions based on work by Burgess, Kroemer and Houston in 1953. In the "21<sup>st</sup> Century" (or "FD") approach, the equation is written

$$J^{\text{MG0}}(\text{FD}) = \{t_{\text{FD}}(f)\}^{-2} \cdot a\phi^{-1}F^2 \exp[-v_{\text{FD}}(f) \cdot b\phi^{3/2}/F], \quad (2)$$

where  $f$  is the "scaled field" defined below.  $v_{\text{FD}}(f)$  and  $t_{\text{FD}}(f)$  are derived, by setting  $x=f$ , from a "purely mathematical" new special mathematical function (SMF)  $v_{\text{FD}}(x)$  introduced by Forbes and Deane in the period 2006 to 2010.  $y_{\text{F}}$  and  $f$  relate to a Schottky-Nordheim tunnelling barrier of zero field height equal to the local work function  $\phi$ , and are given by

$$f = y_{\text{F}}^2 = (e^3/4\pi\epsilon_0) \phi^{-2}F, \quad (3)$$

where  $e$  is the elementary charge and  $\epsilon_0$  is the electrical constant.

The SMF  $v_{\text{FD}}(x)$  is a purely mathematical function that is a special solution of the Euler/Gauss Hypergeometric Differential Equation (HDE) Exact mathematical definitions of  $v_{\text{FD}}(x)$  can be given in many different ways

The "BKH" and "FD" approaches both yield the same numerical results, and currently both are acceptable. However it is argued in this Poster presentation that the FD approach is *mathematically superior* and superior in practice, for the following reasons.

(1) In the precise series expansions for  $v_{\text{FD}}(x)$  there are no terms involving  $x^{1/2}$ : this shows that  $x$  is the natural mathematical variable to use: hence using  $f$  is better than using  $y_{\text{F}}$ .

(2) The defining equation for  $v_{\text{BKH}}(y_{\text{F}})$  is more complicated than the defining equation for  $v_{\text{FD}}(x)$  or  $v_{\text{FD}}(f)$ , and is less obviously a special case of the Euler/Gauss HDE.

(3) Basic algebra related to  $v_{\text{FD}}(f)$  is simpler than basic algebra related to  $v_{\text{BKH}}(y_{\text{F}})$ .

(4) In discussion involving an auxiliary parameter, it is usually easier to work with a parameter that is proportional to "field"  $F$ , rather than with one that is proportional to  $\sqrt{F}$ .

(5) The concept of "scaled field" is easier to understand than the Nordheim parameter.

(6) The orthodoxy test is already formulated in terms of  $f$ , rather than  $y_{\text{F}}$ .

(7) The theory of the Murphy-Good plot (which is more nearly straight than a Fowler-Nordheim plot) is already formulated in terms of  $f$ .

It is argued that, in the long term, it would be better for the subject area if FE researchers changed to using  $f$  rather than  $y_{\text{F}}$ .